**Advanced Algorithms**

**Exercise for Lecture 10**

|  |  |  |  |
| --- | --- | --- | --- |
| **Student Name** |  | **Student ID** |  |
| **Problem 1** |  | | |
| **Problem 2** |  | | |
| **Problem 3** |  | | |
| **Total Score** |  | | |
| **Notes** | Deadline: **2023-10-20 24:00**  Submission Format: ‘**Lecture10\_Name\_Student ID.docx**’, and please send to: **[1914499454@qq.com](mailto:algorithms_23fall@163.com)**.  This assignment is meant to be an evaluation of your **individual** understanding coming into the course and should be completed **without collaboration** or outside help. | | |

1. If i is an integer power of 2, the cost of the i-th operation on a data structure with n is i, otherwise it is 1. Using accounting method to analyze the amortized cost of each operation.
2. Suppose we have a potential function Φ such that Φ(*Di*)≥Φ(*D*0) for all *i*, but Φ(0)≠0 . Show that there exists a potential function Φ′ such that Φ′(D0)=0, Φ′(*Di*)≥0 for all i ≥1, and the amortized costs using Φ′ are the same as the amortized costs using Φ.

1. There are two stacks, A and B, that can perform the following five operations (A size is n, B size is m):

PushA(x): Push x into stack A, actual cost = 1

PushB(x): Push x into stack B, actual cost = 1

MultiPopA(k): Pop up min{k, n} elements from A, actual cost = min{k, n}

MultiPopB(k): Pop up min{k, m} elements from B, actual cost = min{k, m}

Transfer(k): Transfer elements from A to B until transferring k elements or A is empty, actual cost = number of transferred elements

What is the worst-case time complexity of the three operations MultiPopA (k), MultiPopB (k), and Transfer (k)? Define a potential energy function Φ(n, m), which proves that the cost of the above operation is equal to O (1)